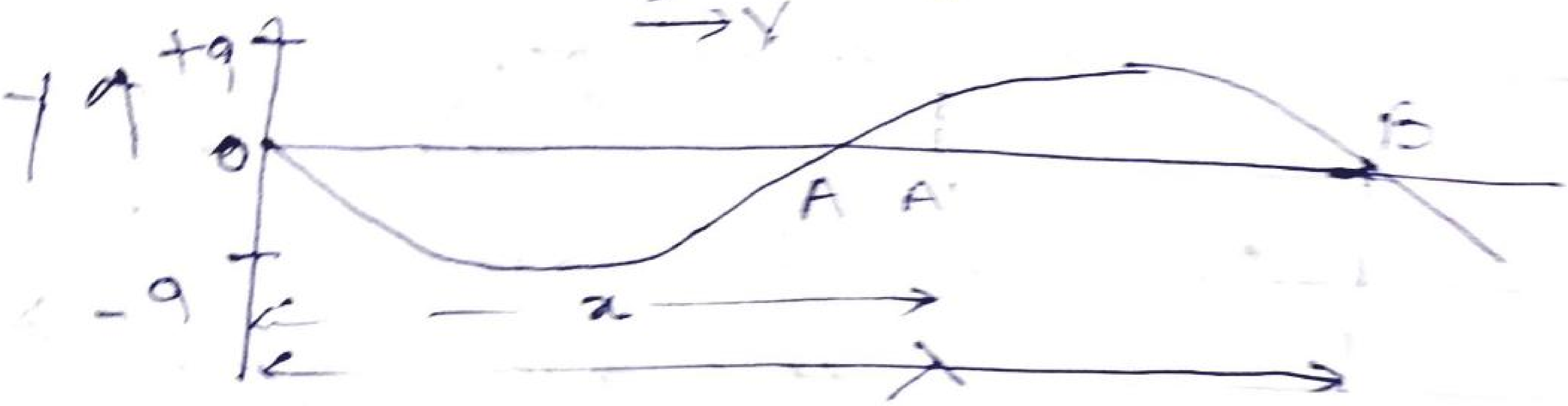
Progressive & Stationary wave

Progressive wave → when the wave in a medium progress from one side to the other is called progressive wave. In such a wave, all the particles of the medium vibrate one by one in the similar way. The transmission of sound in air is longitudinal progressive wave

Equation of progressive wave ⇒



let the wave progresses the particles of the medium performs simple harmonic vibration the displacement of the particle at O at the t time is

$$Y = a \sin\omega t$$

a = amplitude of particle

$$\omega = 2\pi n$$

n = frequency of vibration

P at a distance x from O towards the right

$$y = a\sin(\omega t - \phi)$$

where ϕ = phase difference

λ distance to move = 2π phase

1 " " = $\frac{2\pi}{\lambda}$

x distance travel = $\frac{2\pi x}{\lambda} = \phi$

So $y = a\sin\left(2\pi n t - \frac{2\pi x}{\lambda}\right)$

$= a\sin 2\pi\left(t/T - x/\lambda\right)$

velocity $v = n\lambda$

$v = \lambda/T \quad \frac{1}{T} = \lambda/v$

$= a\sin 2\pi\left(\frac{vt}{\lambda} - x/\lambda\right)$

$= a\sin\frac{2\pi}{\lambda}(vt - x)$

$$\boxed{y = 2\sin\frac{4\pi}{\lambda}(vt - x)}$$

This is equation of progressive wave.

Stationary wave → stationary wave is also called standing wave. When two similar waves travel in a medium is opposite directions in the same line then stati-

Formation of stationary wave

→ When two similar waves travel in opposite direction then by superposition of the two waves stationary waves are formed.

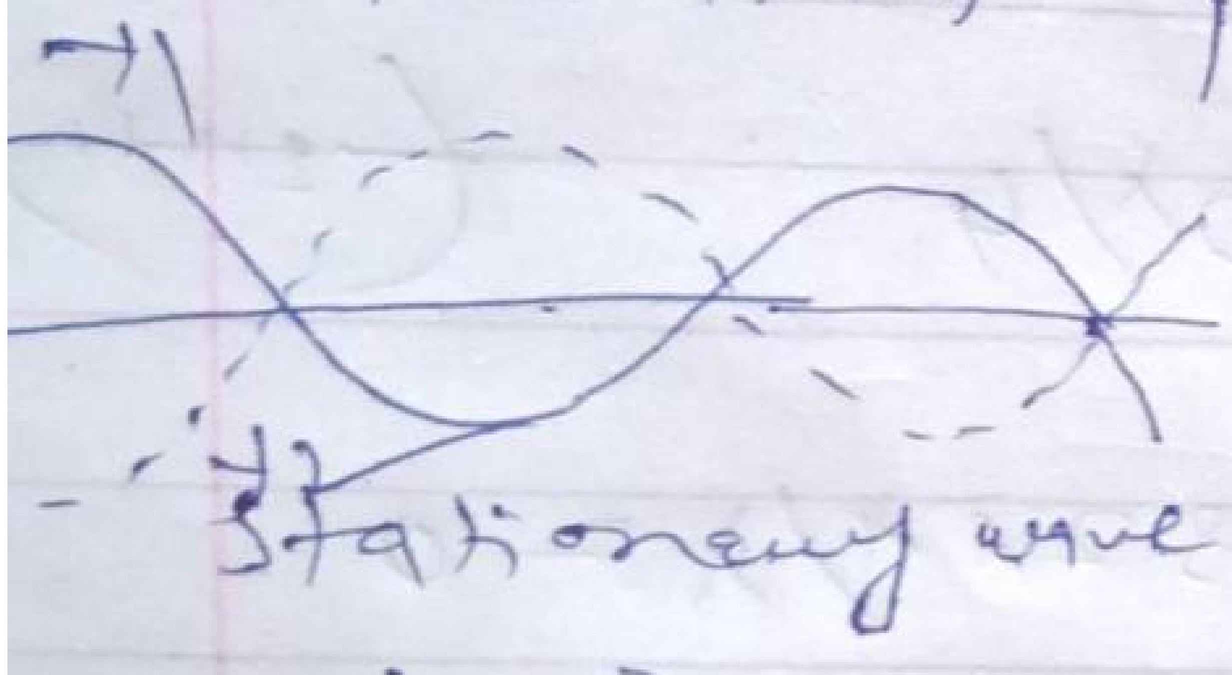
Equation of stationary wave →

let two progressive waves having amplitude a and wavelength λ travel in opposite direction with velocity v

$$Y_1 = a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$Y_2 = a \sin \frac{2\pi}{\lambda}(vt + x)$$

Resultant $Y = Y_1 + Y_2$



$$= a \sin \frac{2\pi}{\lambda}(vt - x) + a \sin \frac{2\pi}{\lambda}(vt + x)$$

$$= a\left[2 \sin \frac{2\pi vt}{\lambda} \cos \frac{2\pi x}{\lambda}\right]$$

$$\therefore Y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda}$$

$$Y = A \sin \frac{2\pi vt}{\lambda}$$

$$A = 2a \cos \frac{2\pi x}{\lambda}$$ Resultant amplitude

condition for maximum amplitude →

$$A = 2a \cos \frac{2\pi x}{\lambda} \text{ max}^n$$

when $\cos \frac{2\pi x}{\lambda} = \pm 1$

$$\frac{2\pi x}{\lambda} = 0, \pi, 2\pi - n\pi$$

$$x = 0, \lambda/2, \frac{2\lambda}{2} --- n\lambda/2$$

$A = \pm 2a$ At antinode.

~~The value of x~~

The distance between any two consecutive antinode $= \lambda/2$

$$A = 2a \cos \frac{2\pi x}{\lambda}$$

$$\cos \frac{2\pi x}{\lambda} = 0$$

$$\frac{2\pi x}{\lambda} = \pi/2, 3\pi/2 - (2n+1)\pi/2$$

$$x = \lambda/4, 3\lambda/4 \quad (2n+1)\lambda/4$$

$$A = 2a \cos 2\pi x/\lambda = 0$$

The distance between any two consecutive nodes and anti nodes is $(2n+1)\lambda/4 - n\lambda/4$

$$= \lambda/4$$

Characteristics of stationary wave →

(1) Neither the disturbance proceeds forward nor the particles impart their motion between themselves.

(2) The amplitude at the different points of the medium is different. maximum at antinodes and minimum at ~~at~~ nodes.

(3) Between two nodes all the points are in the same phase of vibration.

(4) At node the change of pressure and density is maximum and at antinodes the change is minimum.